TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 4 2022

Section 1 Calculator Free

Trigonometric Functions

STUDENT'S NAME MARKING KEY [KRISZYK]

DATE: Wednesday 3rd August **TIME:** 30 minutes **MARKS**: 34

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, approved Formula sheet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

Evaluate $\cos\left(\frac{5\pi}{12}\right)\sin\left(\frac{7\pi}{12}\right)$ $\cos A \sin B$

$= \frac{1}{2}\left[\sin\left(\frac{5\pi}{12}+\frac{7\pi}{12}\right) - \sin\left(\frac{5\pi}{12}-\frac{7\pi}{12}\right)\right]$ ✓

$= \frac{1}{2}\left[\sin\left(\frac{12\pi}{12}\right) - \sin\left(\frac{-2\pi}{12}\right)\right]$ ✓

$= \frac{1}{2}\left[\sin\pi - \sin\left(\frac{-\pi}{6}\right)\right]$

$= \frac{1}{2}\left[0 - \left(-\frac{1}{2}\right)\right]$

$= \frac{1}{4}$ ✓

2. (11 marks)

(a) Solve $\cos\left(2x - \frac{\pi}{4}\right) = \frac{1}{2}$ [3]

$2x - \frac{\pi}{4} = \frac{\pi}{3} + 2\pi k$ ✓

$-\frac{\pi}{3} + 2\pi k$

$2x = \frac{4\pi + 24\pi k + 3\pi}{12}$

$\frac{-4\pi + 24\pi k + 3\pi}{12}$

$2x = \frac{7\pi + 24\pi k}{12}, \frac{-\pi + 24\pi k}{12}$ ✓

$x = \frac{7\pi + 24\pi k}{24}, \frac{-\pi + 24\pi k}{24}$ ✓

where $k \in \mathbb{Z}$

(b) Solve $\cos(2\theta) - \cos\theta = 0$ for $0 \leq x \leq 2\pi$ [4]

$2\cos^2\theta - 1 - \cos\theta = 0$ ✓

$(2\cos\theta + 1)(\cos\theta - 1) = 0$ ✓

$\cos\theta = \frac{-1}{2}$ $\cos\theta = 1$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ $\theta = 0, 2\pi$

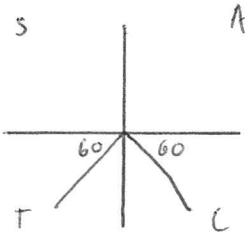
$\therefore \theta = 0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi$ ✓✓

(c) Solve $3\operatorname{cosec} 2\theta = -2\sqrt{3}$ for $-180° \leq x \leq 180°$ [4]

$\operatorname{cosec} 2\theta = \frac{-2\sqrt{3}}{3}$ ✓

$\operatorname{cosec} 2\theta = \frac{-2}{\sqrt{3}}$

$\therefore \quad \sin 2\theta = -\frac{\sqrt{3}}{2}$ ✓



$2\theta = -60, -120, 300, 240$

$\theta = -30°, -60°, 150°, 120°$ ✓✓

3. (6 marks)

Prove the following.

(a) $\dfrac{1}{1+\tan^2\theta} = \cos^2\theta$ [2]

$$LHS = \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}} \quad \checkmark \qquad = \frac{\cos^2\theta}{\cos^2\theta+\sin^2\theta} \quad \checkmark$$

$$= \frac{1}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}} \qquad = \frac{\cos^2\theta}{1}$$

$$= RHS \qquad Q.E.D.$$

(b) $\cos(P+Q)\cos(P-Q) = \cos^2 P + \cos^2 Q - 1$ [4]

$$RHS = (\cos P\cos Q - \sin P\sin Q)(\cos P\cos Q + \sin P\sin Q) \quad \checkmark$$

$$= \cos^2 P\cos^2 Q - \sin^2 P\sin^2 Q \quad \checkmark$$

$$= \cos^2 P\cos^2 Q - (1-\cos^2 P)(1-\cos^2 Q)$$

$$= \cos^2 P\cos^2 Q - (1-\cos^2 P-\cos^2 Q+\cos^2 P\cos^2 Q)$$

$$= -1+\cos^2 P+\cos^2 Q$$

$$= LHS$$

$$Q.E.D.$$

4. (8 marks)

Prove the following.

(a) $\cot\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)=2\operatorname{cosec}(x)$ [4]

$$LHS = \frac{\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)} + \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} \checkmark$$

$$= \frac{\cos^2\left(\frac{x}{2}\right) + \sin^2\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}$$

$$= \frac{1}{\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)} \checkmark$$

$$= \frac{1}{\frac{1}{2}\sin 2\left(\frac{x}{2}\right)}$$

$$= 2 \times \frac{1}{\sin(x)}$$

$$= 2\operatorname{cosec}(x)$$

$$= RHS \quad Q.E.D.$$

(b) $1 + 2\operatorname{cosec}\theta\cot\theta + 2\cot^2\theta = \frac{1+\cos\theta}{1-\cos\theta}$ [4]

$$LHS = 1 + \frac{2\cos\theta}{\sin\theta\sin\theta} + 2\frac{\cos^2\theta}{\sin^2\theta} \checkmark$$

$$= \frac{\sin^2\theta + 2\cos\theta + 2\cos^2\theta}{\sin^2\theta} \checkmark$$

$$= \frac{(1-\cos^2\theta) + 2\cos\theta + 2\cos^2\theta}{1-\cos^2\theta} \checkmark$$

$$= \frac{1 + 2\cos\theta + \cos^2\theta}{1-\cos^2\theta}$$

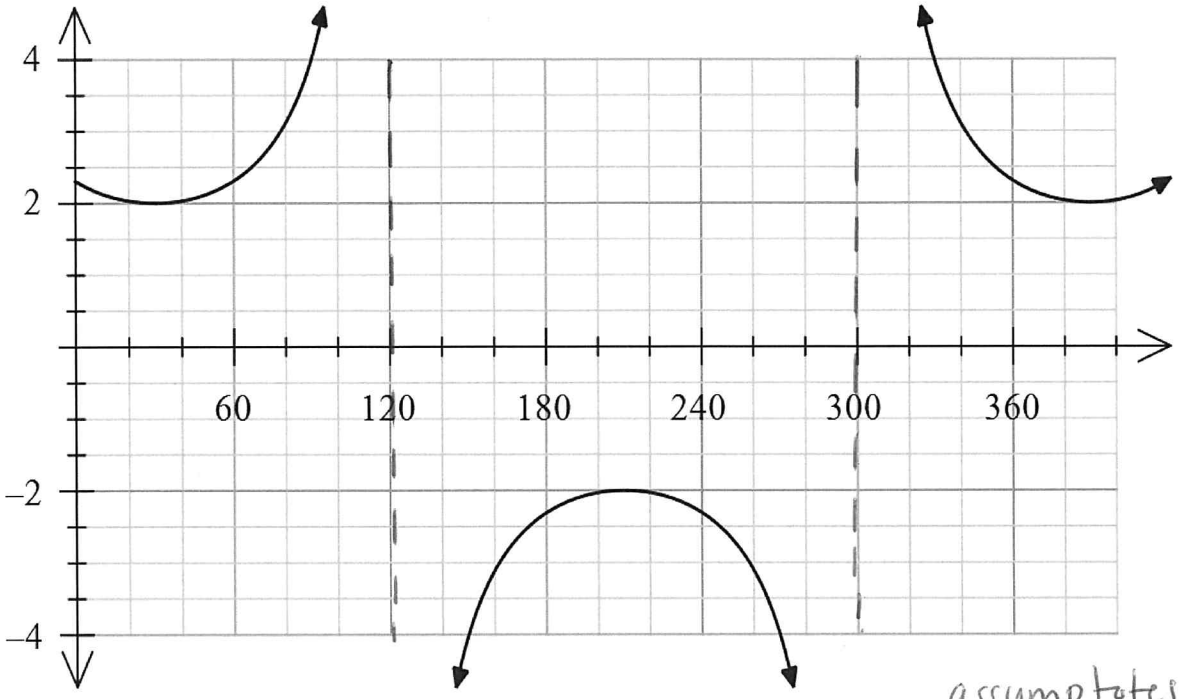
$$= RHS$$

$$= \frac{(1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} \checkmark$$

QED

5. (6 marks)

(a) Sketch $f(x) = 2\sec(x - 30°)$ on the graph below. [3]



assymptotes ✓
shape ✓
min/max ✓

(b) Simplify $\dfrac{\sin 6B + \sin 2B}{\sin 6B - \sin 2B}$ [3]

$$= \frac{2\sin\left(\frac{6B+2B}{2}\right)\cos\left(\frac{6B-2B}{2}\right)}{2\cos\left(\frac{6B+2B}{2}\right)\sin\left(\frac{6B-2B}{2}\right)} \quad \checkmark$$

$$= \frac{\sin(4B)\cos(2B)}{\cos(4B)\sin(2B)} \quad \checkmark$$

$$= \tan(4B)\cot(2B) \quad \checkmark$$

Extra working page

Q ____

TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 1 2022

Section 2 Calculator Assumed

Trigonometric Functions

STUDENT'S NAME MARKING KEY [KRISZYK]

DATE: Wednesday 3rd August **TIME:** 15 minutes **MARKS**: 16

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, approved Formula sheet

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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6. (5 marks)

Let the angle $\theta = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$.

(a) Use your calculator to determine an exact value for $\sin\left(\frac{\pi}{12}\right)$. [1]

$$\frac{\sqrt{2}\left(\sqrt{3}-1\right)}{4}$$

(b) Use an angle sum or difference identity to show how to obtain the above exact value for $\sin\left(\frac{\pi}{12}\right)$. [4]

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \quad \checkmark$$

$$= \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) \quad \checkmark$$

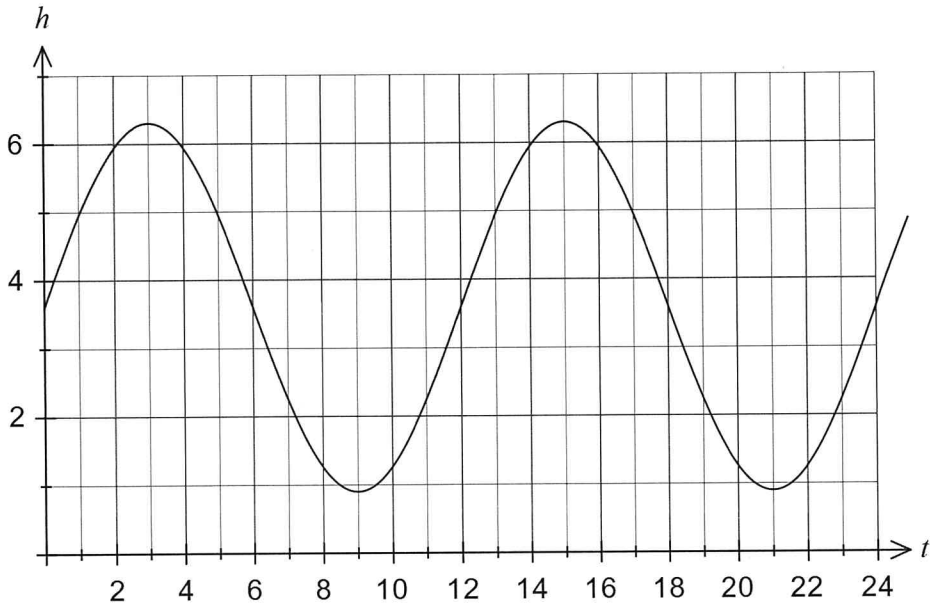
$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} \quad \checkmark$$

$$= \frac{\sqrt{2}\left(\sqrt{3}-1\right)}{4} \quad \checkmark$$

7. (5 marks)

The clearance, h metres, under a bridge spanning a river estuary varies with the time since midnight, t hours, and is given by $h = 3.6 + 2.7\ sin\left(\frac{\pi t}{6}\right)$.

(a) Sketch the graph of the clearance against time on the axes below. [3]



(b) Determine the percentage of any 24-hour period during which the clearance under the bridge is no more than two metres. [2]

solve height ≤ 2

$t = 7.211,\ 10.788,\ 19.211,\ 22.788$ ✓

$$\frac{10.788 - 7.211 + 22.788 - 19.211}{24} \times 100$$

$= 29.8\%$

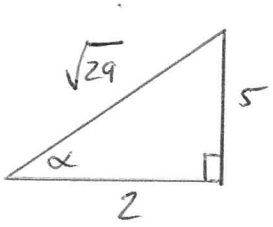
8. (6 marks)

Consider the function $f(t) = 2\sin t - 5\cos t,\ t \geq 0$.

(a) Express $f(t)$ in the form $r\sin(t-\alpha)$, where $r > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$ and state the values of r and α, rounded to 2 decimal places. [4]

$= 2\sin t - 5\cos t$ ✓

$= \sqrt{29}\left(\frac{2}{\sqrt{29}}\sin t - \frac{5}{\sqrt{29}}\cos t\right)$ ✓



$= \sqrt{29}\sin(t-\alpha)$ ✓

$= \sqrt{29}\sin(t-1.19^r)$ ✓

-1 degrees

(b) Hence or otherwise determine the minimum value of $f(t)$ and the smallest value of t for this minimum to occur. [2]

min value $= -\sqrt{29}$

~ -5.39 ✓

occurs @ $\frac{3\pi}{2} + 1.19^r$

$\sim 5.90^r$ ✓